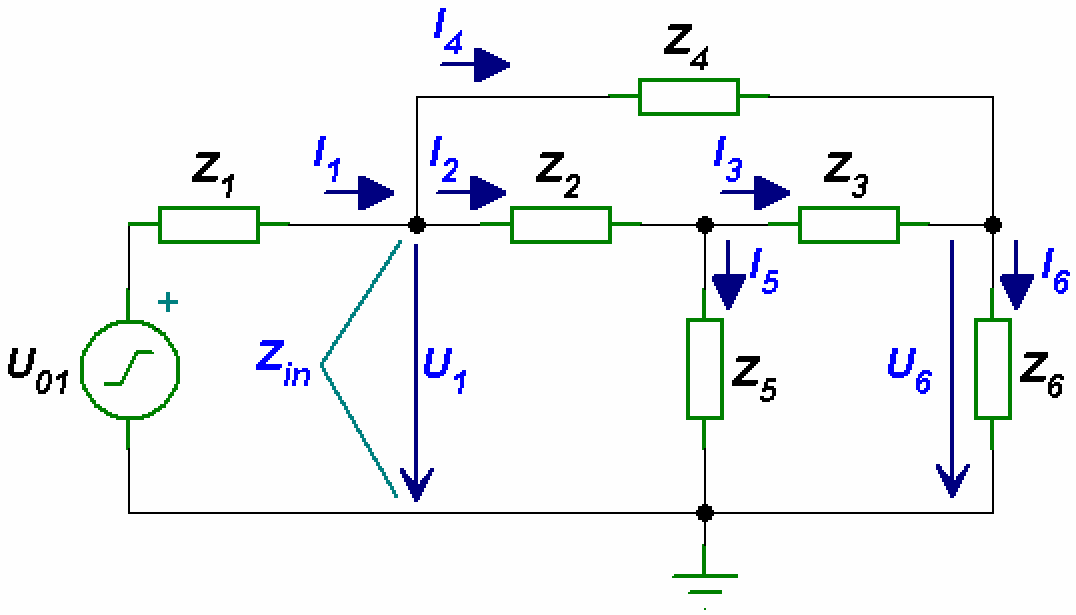
Loop Analysis - Incidence Matrix B

Example

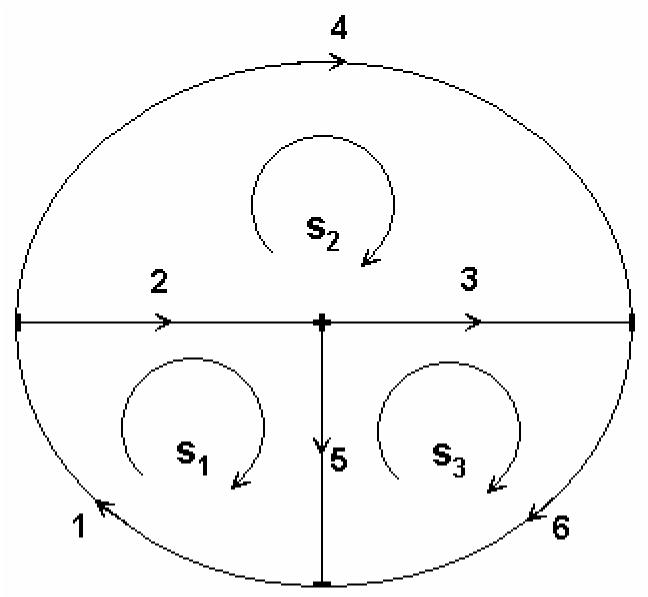
For the circuit shown below find the currents I_1 to I_6, the voltage U_6, the input impedance Z_{in} and the voltage transfer $K_{06} = U_6 / U_{01}$. Use incidence matrix **B** to solve this task, when $Z_1 = Z_6 = 50\ \Omega$, $Z_2 = Z_3 = -j\ 50\ \Omega$, $Z_4 = Z_5 = j\ 100\ \Omega$, $u_{01}(t) = 10 \cos(\omega t)$ V and $\omega = 10^5$ rad/s. Z_1 is the internal resistance of the source.



Circuit

Solution

For the voltage source given by $u_{01}(t) = 10 \cos(\omega t)$ the phasor is $U_{01} = 10\ e^{j\,0°} = 10\angle 0°$.



Circuit Diagram

For the circuit diagram shown above we get the incidence matrix **B**

$$\mathbf{B} = \begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 & \\ & \left[\begin{matrix} 1 \\ 0 \\ 0 \end{matrix}\right. & \begin{matrix} 1 \\ -1 \\ 0 \end{matrix} & \begin{matrix} 0 \\ -1 \\ 1 \end{matrix} & \begin{matrix} 0 \\ 1 \\ 0 \end{matrix} & \begin{matrix} 1 \\ 0 \\ -1 \end{matrix} & \left.\begin{matrix} 0 \\ 0 \\ 1 \end{matrix}\right] & \begin{matrix} s_1 \\ s_2 \\ s_3 \end{matrix} \end{matrix}$$

The matrix $\mathbf{U_0}$ of the branch sources and the branch impedance matrix **Z** are

$$\mathbf{U_0} = \begin{bmatrix} \boldsymbol{U_{01}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{Z} = \begin{bmatrix} \boldsymbol{Z_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{Z_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \boldsymbol{Z_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \boldsymbol{Z_4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \boldsymbol{Z_5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \boldsymbol{Z_6} \end{bmatrix}$$

The matrix of the loop voltage sources is

$$\mathbf{U_{0S}} = \mathbf{B}\ \mathbf{U_0}$$

The loop impedance matrix is

$$\mathbf{Z_S = B\ Z\ {}^{T}B} \qquad (\ {}^{T}\mathbf{B} \text{ is the transpose of the matrix } \mathbf{B})$$

The loop current matrix is

$$\mathbf{I_S} = \mathbf{Z_S^{-1}}\ \mathbf{U_{0S}} \qquad (\mathbf{Z_S^{-1}} \text{ is the inverse of the matrix } \mathbf{Z_S})$$

The branch current matrix is

$$\mathbf{I} = {}^{T}\mathbf{B}\ \mathbf{I_S}$$

The input impedance is

$$\boldsymbol{Z_{\mathrm{in}}} = \frac{\boldsymbol{U_1}}{\boldsymbol{I_1}} = \frac{\boldsymbol{U_{01}}}{\boldsymbol{I_1}} - \boldsymbol{Z_1} \qquad (\boldsymbol{Z_1 I_1 + U_1 = U_{01}} \quad \Rightarrow \quad \boldsymbol{U_1 = U_{01} - Z_1 I_1})$$

The voltage $\boldsymbol{U_6}$ and the voltage transfer $\boldsymbol{K_{06}}$ are

$$\boldsymbol{U_6 = Z_6 I_6} \qquad \boldsymbol{K_{06}} = \frac{\boldsymbol{U_6}}{\boldsymbol{U_{01}}}$$

The MATLAB program for solving this task is

MATLAB Script

```
clear; clc
% input values
% the impedances are in ohms
Z1=50; Z2=-j*50; Z3=-j*50; Z4=j*100; Z5=j*100; Z6=50;
% the voltages are in volts
% voltage u01:
u01max=10; u01angle=0; % angle in degrees
% complex representation of the voltage u01
U01=u01max*exp(j*u01angle*pi/180);
% incidence matrix B
B=[1  1  0  0  1  0;
   0 -1 -1  1  0  0;
   0  0  1  0 -1  1];
% column matrix U0
U0=[U01; 0; 0; 0; 0; 0];
% diagonal matrix Z
Z=diag([Z1 Z2 Z3 Z4 Z5 Z6]);
U0s=B*U0;
Zs=B*Z*B';
Is=inv(Zs)*U0s;
I=B'*Is
Zin=U01/I(1)-Z1
U6=I(6)*Z6
Ku=U6/U01
```

The results obtained from MATLAB are

```
I =
   0.0100 - 0.0300i
   0.0100 + 0.0700i
  -0.0100 + 0.1300i
        0 - 0.1000i
   0.0200 - 0.0600i
  -0.0100 + 0.0300i

Zin =
  5.0000e+001 +3.0000e+002i

U6 =
  -0.5000 + 1.5000i

Ku =
  -0.0500 + 0.1500i
```