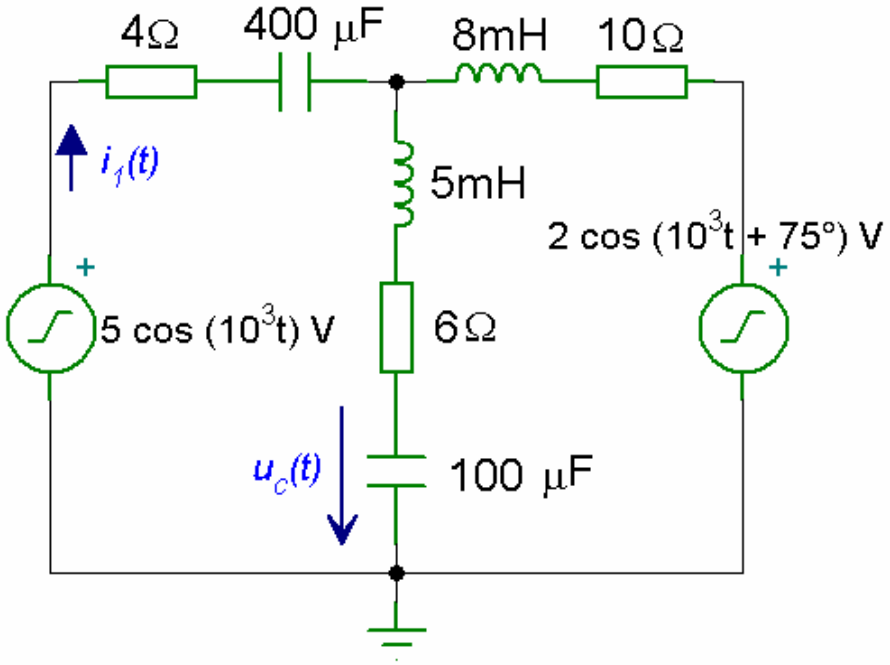
Loop Analysis (AC Analysis)

Example

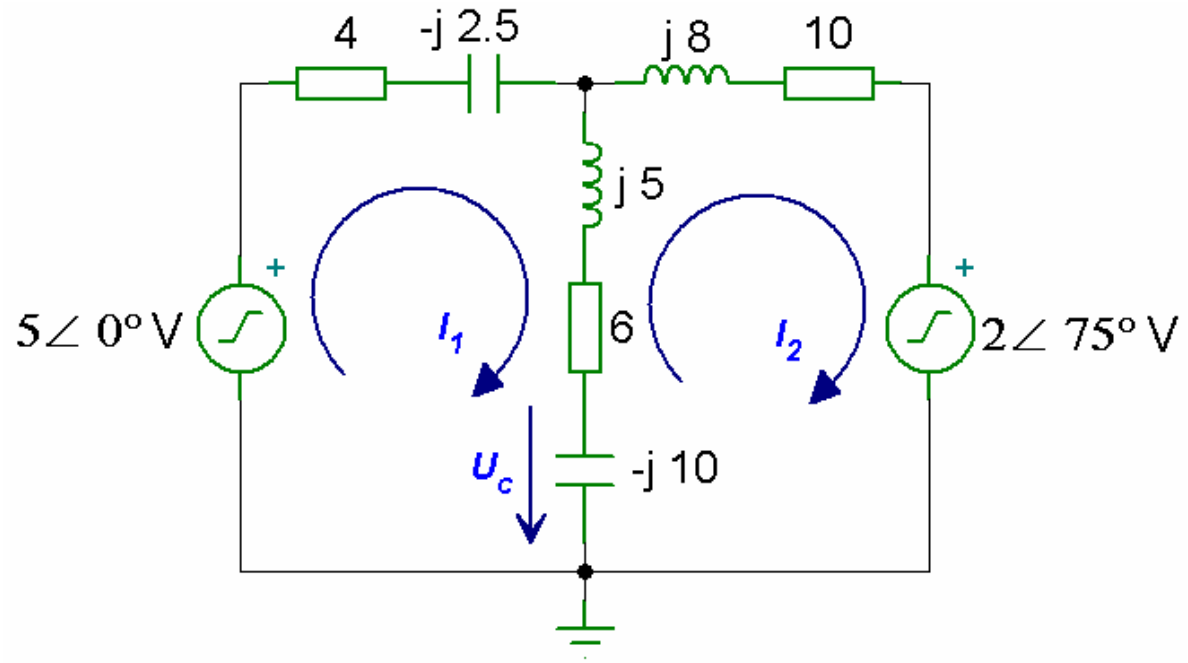
For the circuit shown below find the current $i_1(t)$ and the voltage $u_C(t)$.



Circuit with Two Sources – Time Domain

Solution

Using loop analysis and complex frequency representation we obtain the following figure. The impedances are in ohms.



Circuit with Two Sources – Frequency Domain Equivalent

The loop equations are:

Loop 1: $-5\angle 0° + (4 - j\,2.5)\,\boldsymbol{I}_1 + (6 + j\,5 - j\,10)\,(\boldsymbol{I}_1 - \boldsymbol{I}_2) = 0$

$$(10 - j\,7.5)\,\boldsymbol{I}_1 + (-6 + j\,5)\,\boldsymbol{I}_2 = 5\angle 0°$$

Loop 2: $(10 + j\,8)\,\boldsymbol{I}_2 + 2\angle 75° + (6 + j\,5 - j\,10)\,(\boldsymbol{I}_2 - \boldsymbol{I}_1) = 0$

$$(-6 + j\,5)\,\boldsymbol{I}_1 + (16 + j\,3)\,\boldsymbol{I}_2 = -2\angle 75°$$

In matrix form, we have

$$\begin{bmatrix} 10 - j7.5 & -6 + j5 \\ -6 + j5 & 16 + j3 \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix} = \begin{bmatrix} 5\angle 0° \\ -2\angle 75° \end{bmatrix}$$

The voltage $\boldsymbol{U}_C$ can be obtained as

$$\boldsymbol{U}_C = -j\,10\,(\boldsymbol{I}_1 - \boldsymbol{I}_2)$$

The MATLAB program for solving the loop currents $\boldsymbol{I}_1$, $\boldsymbol{I}_2$ and the voltage $\boldsymbol{U}_C$ is

MATLAB Script

```
% This program calculates the phasor current I1 and
% phasor voltage Uc.
% impedance matrix
Z = [10-j*7.5   -6+j*5;
     -6+j*5     16+j*3];
% voltage vector
U = [5; -2*exp(j*75*pi/180)];
I = inv(Z)*U; % solution for loop currents
Uc = -10*j*(I(1)-I(2));
I1_abs = abs(I(1));
I1_ang = angle(I(1))*180/pi;
Uc_abs = abs(Uc);
Uc_ang = angle(Uc)*180/pi;
disp('current I1:')
disp(['magnitude = ',num2str(I1_abs),' A'])
disp(['angle = ',num2str(I1_ang),'°'])
disp('_____________________________')
disp('voltage Uc:')
disp(['magnitude = ',num2str(Uc_abs),' V'])
disp(['angle = ',num2str(Uc_ang),'°'])
```

The results obtained from MATLAB are

```
current I1:
magnitude = 0.38771 A
angle = 15.0193°

_____________________________
voltage Uc:
magnitude = 4.2183 V
angle = -40.8617°
```

From the MATLAB results, the time domain current $i_1(t)$ is

$$i_1(t) = 0.388 \cos (10^3 t + 15.02°) \text{ A}$$

and the time domain voltage $u_C(t)$ is

$$u_C(t) = 4.218 \cos (10^3 t - 40.86°) \text{ V}$$