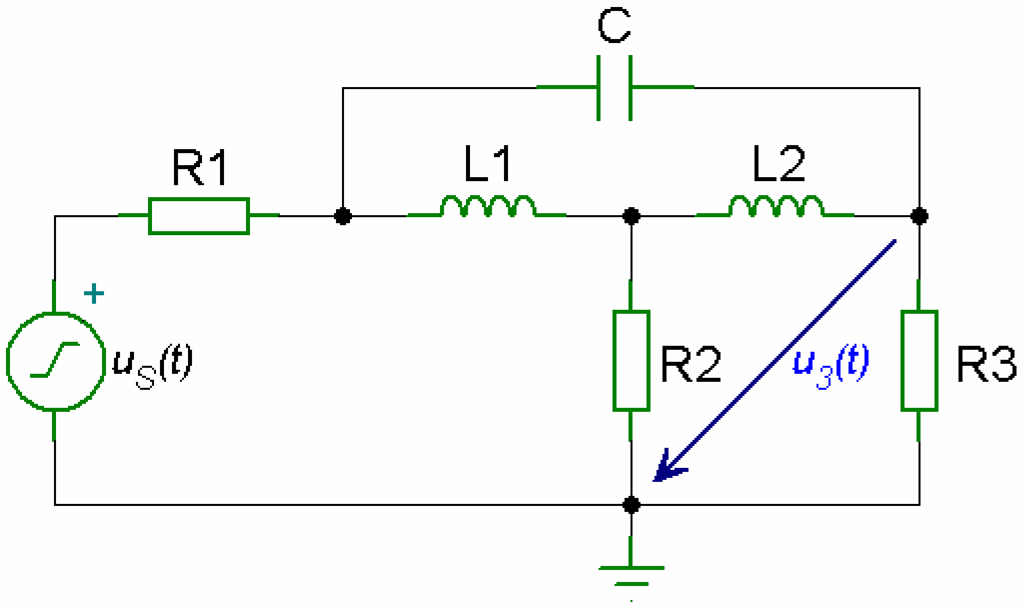
Nodal Analysis (AC Analysis)

Example

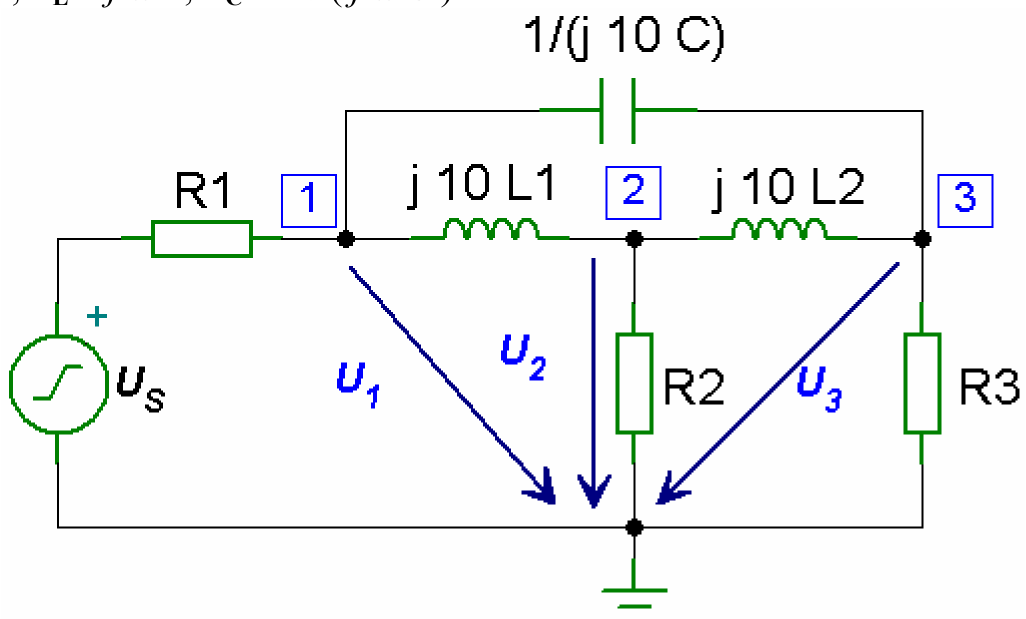
For the circuit shown below find the voltage $u_3(t)$ when $R_1 = 20\ \Omega$, $R_2 = 100\ \Omega$, $R_3 = 50\ \Omega$, $L_1 = 4$ H, $L_2 = 8$ H, $C = 250\ \mu$F, $u_S(t) = 8\cos(\omega t + 15°)$ V and $\omega = 10$ rad/s.



RLC Circuit with Sinusoidal Excitation – Time Domain

Solution

Using nodal analysis and complex frequency representation, the analysis can be simplified. For the voltage source given by $u_S(t) = 8\cos(\omega t + 15°)$ the phasor is $\boldsymbol{U}_S = 8\, e^{j\,15°} = 8\angle 15°$. For the components R, L, C the complex impedances are $\boldsymbol{Z}_R = R$, $\boldsymbol{Z}_L = j\,\omega L$, $\boldsymbol{Z}_C = 1 / (j\,\omega C)$.



RLC Circuit with Sinusoidal Excitation – Frequency Domain Equivalent

Using nodal analysis, we obtain the following equations.

For node 1:

$$-\frac{\boldsymbol{U}_{\mathrm{S}}-\boldsymbol{U}_{\mathbf{1}}}{R_1}+\frac{\boldsymbol{U}_{\mathbf{1}}-\boldsymbol{U}_{\mathbf{2}}}{j10L_1}+\frac{\boldsymbol{U}_{\mathbf{1}}-\boldsymbol{U}_{\mathbf{3}}}{\frac{1}{j10C}}=0$$

$$-\frac{\boldsymbol{U}_{\mathrm{S}}-\boldsymbol{U}_{\mathbf{1}}}{20}-j\frac{\boldsymbol{U}_{\mathbf{1}}-\boldsymbol{U}_{\mathbf{2}}}{40}+j0.0025\left(\boldsymbol{U}_{\mathbf{1}}-\boldsymbol{U}_{\mathbf{3}}\right)=0$$

$$0.05\ \boldsymbol{U}_{\mathbf{1}}-j\ 0.025\ \boldsymbol{U}_{\mathbf{1}}+j\ 0.025\ \boldsymbol{U}_{\mathbf{2}}+j\ 0.0025\ \boldsymbol{U}_{\mathbf{1}}-j\ 0.0025\ \boldsymbol{U}_{\mathbf{3}}=0.05\ \boldsymbol{U}_{\mathrm{S}}$$

$$(0.05-j\ 0.0225)\ \boldsymbol{U}_{\mathbf{1}}+j\ 0.025\ \boldsymbol{U}_{\mathbf{2}}-j\ 0.0025\ \boldsymbol{U}_{\mathbf{3}}=0.4\angle\ 15^\circ$$

At node 2:

$$-\frac{\boldsymbol{U}_{\mathbf{1}}-\boldsymbol{U}_{\mathbf{2}}}{j10L_1}+\frac{\boldsymbol{U}_{\mathbf{2}}}{R_2}+\frac{\boldsymbol{U}_{\mathbf{2}}-\boldsymbol{U}_{\mathbf{3}}}{j10L_2}=0$$

$$j\frac{\boldsymbol{U}_{\mathbf{1}}-\boldsymbol{U}_{\mathbf{2}}}{40}+\frac{\boldsymbol{U}_{\mathbf{2}}}{100}-j\frac{\boldsymbol{U}_{\mathbf{2}}-\boldsymbol{U}_{\mathbf{3}}}{80}=0$$

$$j\ 0.025\ \boldsymbol{U}_{\mathbf{1}}-j\ 0.025\ \boldsymbol{U}_{\mathbf{2}}+0.01\ \boldsymbol{U}_{\mathbf{2}}-j\ 0.0125\ \boldsymbol{U}_{\mathbf{2}}+j\ 0.0125\ \boldsymbol{U}_{\mathbf{3}}=0$$

$$j\ 0.025\ \boldsymbol{U}_{\mathbf{1}}+(0.01-j\ 0.0375)\ \boldsymbol{U}_{\mathbf{2}}+j\ 0.0125\ \boldsymbol{U}_{\mathbf{3}}=0$$

At node 3:

$$-\frac{\boldsymbol{U}_{\mathbf{1}}-\boldsymbol{U}_{\mathbf{3}}}{\frac{1}{j10C}}-\frac{\boldsymbol{U}_{\mathbf{2}}-\boldsymbol{U}_{\mathbf{3}}}{j10L_2}+\frac{\boldsymbol{U}_{\mathbf{3}}}{R_3}=0$$

$$-j0.0025\left(\boldsymbol{U}_{\mathbf{1}}-\boldsymbol{U}_{\mathbf{3}}\right)+j\frac{\boldsymbol{U}_{\mathbf{2}}-\boldsymbol{U}_{\mathbf{3}}}{80}+\frac{\boldsymbol{U}_{\mathbf{3}}}{50}=0$$

$$-j\ 0.0025\ \boldsymbol{U}_{\mathbf{1}}+j\ 0.0025\ \boldsymbol{U}_{\mathbf{3}}+j\ 0.0125\ \boldsymbol{U}_{\mathbf{2}}-j\ 0.0125\ \boldsymbol{U}_{\mathbf{3}}-0.02\ \boldsymbol{U}_{\mathbf{3}}=0$$

$$-j\ 0.0025\ \boldsymbol{U}_{\mathbf{1}}+j\ 0.0125\ \boldsymbol{U}_{\mathbf{2}}+(0.02-j\ 0.01)\ \boldsymbol{U}_{\mathbf{3}}=0$$

We get the matrix equation

$$\begin{bmatrix} 0.05 - j0.0225 & j0.025 & -j0.0025 \\ j0.025 & 0.01 - j0.0375 & j0.0125 \\ -j0.0025 & j0.0125 & 0.02 - j0.01 \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_1 \\ \boldsymbol{U}_2 \\ \boldsymbol{U}_3 \end{bmatrix} = \begin{bmatrix} 0.4\angle 15^\circ \\ 0 \\ 0 \end{bmatrix}$$

The MATLAB program for solving voltage $\boldsymbol{U}_3$ is

MATLAB Script

```
% This program computes the nodal voltage U3
% Y is the admittance matrix
% Us is the voltage source phasor
% I is the current matrix
% U is the voltage vector
Y = [0.05-j*0.0225   j*0.025   -j*0.0025;
     j*0.025      0.01-j*0.0375  j*0.0125;
     -j*0.0025     j*0.0125   0.02-j*0.01];
Us = 8*exp(j*15*pi/180);
I = [0.05*Us; 0; 0];
% solution for nodal voltages
U = inv(Y)*I;
% magnitude of nodal voltage U3
U3_abs = abs(U(3));
% angle of nodal voltage U3 in degrees
U3_ang = angle(U(3))*180/pi;
disp('voltage U3:')
disp(['magnitude = ',num2str(U3_abs),' V'])
disp(['angle = ',num2str(U3_ang),'°'])
```

The results obtained from MATLAB are

```
voltage U3:
magnitude = 1.8504 V
angle = -72.4533°
```

From the MATLAB results, the time domain voltage $u_3(t)$ is

$$u_3(t) = 1.85 \cos(\omega t - 72.45^\circ) \text{ V}$$