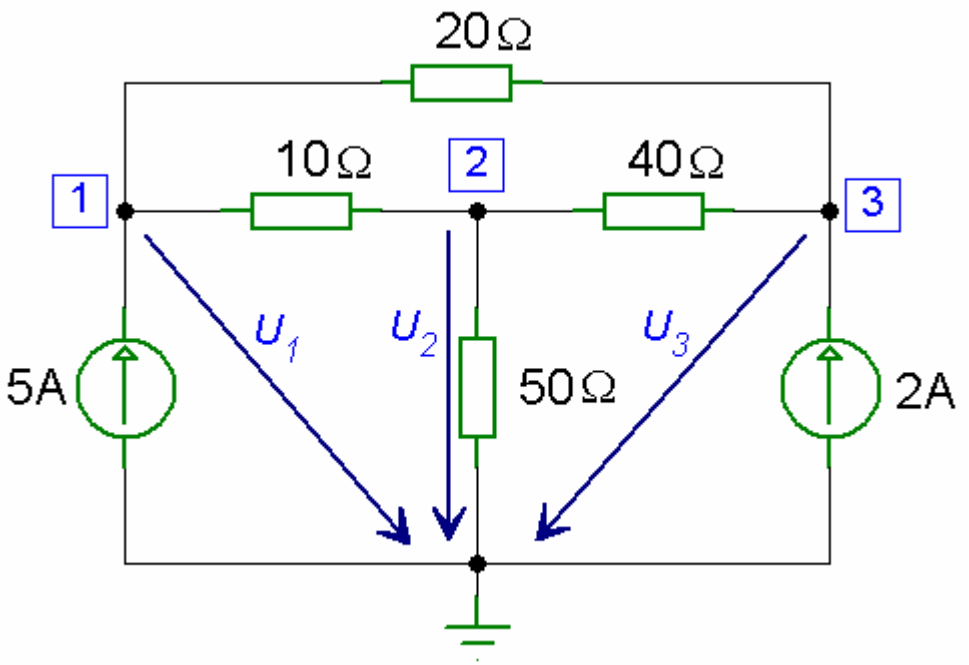
Nodal Analysis

Example

For the circuit shown below find the nodal voltages U_1, U_2 and U_3.



Circuit with Nodal Voltages

Solution

Using Kirchhoff's current law and assuming that the currents leaving a node are positive, we have

For node 1:

$$\frac{U_1 - U_2}{10} + \frac{U_1 - U_3}{20} - 5 = 0$$

$$0.1\ U_1 - 0.1\ U_2 + 0.05\ U_1 - 0.05\ U_3 - 5 = 0$$

$$0.15\ U_1 - 0.1\ U_2 - 0.05\ U_3 = 5$$

At node 2:

$$-\frac{U_1 - U_2}{10} + \frac{U_2}{50} + \frac{U_2 - U_3}{40} = 0$$

$$-0.1\ U_1 + 0.1\ U_2 + 0.02\ U_2 + 0.025\ U_2 - 0.025\ U_3 = 0$$

$$-0.1\ U_1 + 0.145\ U_2 - 0.025\ U_3 = 0$$

At node 3:

$$-\frac{U_1 - U_3}{20} - \frac{U_2 - U_3}{40} - 2 = 0$$

$$-0.05\,U_1 + 0.05\,U_3 - 0.025\,U_2 + 0.025\,U_3 - 2 = 0$$

$$-0.05\,U_1 - 0.025\,U_2 + 0.075\,U_3 = 2$$

In matrix form, we have

$$\begin{bmatrix} 0.15 & -0.1 & -0.05 \\ -0.1 & 0.145 & -0.025 \\ -0.05 & -0.025 & 0.075 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$$

The MATLAB program for solving the nodal voltages is

MATLAB Script

```
% this program computes the nodal voltages
% given the admittance matrix Y and current vector I
% Y is the admittance matrix
% I is the current vector
% U is the nodal voltage vector
% initialize matrix Y and vector I using YU=I form
Y = [ 0.15    -0.1     -0.05;
     -0.1      0.145   -0.025;
     -0.05    -0.025    0.075];
I = [5; 0; 2];
% solution for the voltages
disp('The nodal voltages U1, U2 and U3 are')
U = inv(Y)*I
```

The results obtained from MATLAB are

```
The nodal voltages U1, U2 and U3 are

U =

  404.2857
  350.0000
  412.8571
```