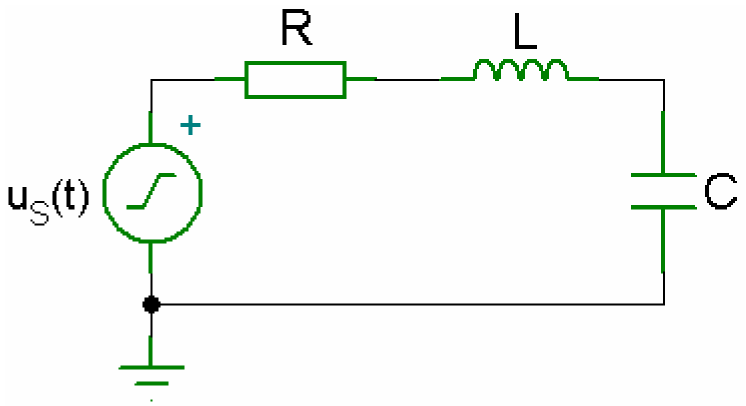
Phasor Diagram

Example

For the circuit shown below with using MATLAB plot the phasor diagram and find the complex power $\boldsymbol{S}$, the average power P, the reactive power Q and the power factor $\cos\varphi$ when R = 5 Ω, L = 20 mH, C = 400 μF and $u_S(t) = 100 \sin(314\, t + 20°)$ V.



RLC Circuit

Solution

Using complex frequency representation, for the voltage source $u_S(t)$ the phasor is $\boldsymbol{U}_S = 100\ e^{j\,20°} = 100\angle 20°$. For the components R, L, C the complex impedances are

$$\boldsymbol{Z}_R = R = 5\ \Omega$$

$$\boldsymbol{Z}_L = j\,\omega L = j\,314 \cdot 0.020 = j\,6.28 = 6.28\angle 90°\ \Omega$$

$$\boldsymbol{Z}_C = 1 / (j\,\omega C) = 1 / (j\,314 \cdot 400 \cdot 10^{-6}) = 1 / (j\,0.1256) =$$

$$= -j\,7.9618 = 7.9618\angle -90°\ \Omega$$

The total complex impedance is

$$\boldsymbol{Z} = \boldsymbol{Z}_R + \boldsymbol{Z}_L + \boldsymbol{Z}_C = 5 - j\,1.6818 = 5.2753\angle -18.59°\ \Omega$$

The current is

$$\boldsymbol{I} = \frac{\boldsymbol{U}_S}{\boldsymbol{Z}} = \frac{100\angle 20°}{5.275\angle -18.57°} = 18.9564\angle 38.59°\ \text{A}$$

For the components R, L, C the voltages are

$$\boldsymbol{U}_{\mathbf{R}} = \boldsymbol{Z}_{\mathbf{R}}\, \boldsymbol{I} = 94.782\angle\, 38.59°\ \text{V}$$

$$\boldsymbol{U}_{\mathbf{L}} = \boldsymbol{Z}_{\mathbf{L}}\, \boldsymbol{I} = 119.0462\angle\, 128.59°\ \text{V}$$

$$\boldsymbol{U}_{\mathbf{C}} = \boldsymbol{Z}_{\mathbf{C}}\, \boldsymbol{I} = 150.9268\angle -51.41°\ \text{V}$$

The power factor is

$$\cos\varphi = \cos(\alpha_{\text{U}} - \alpha_{\text{I}}) = \cos(20° - 38.59°) = \cos(-18.59°) = 0.948$$

where α_{U} and α_{I} are the phase angles of the $\boldsymbol{U}_{\text{S}}$ and $\boldsymbol{I}$ phasors

The complex power is

$$\boldsymbol{S} = \boldsymbol{U}_{\mathbf{RMS}}\, \boldsymbol{I}_{\mathbf{RMS}}^{*} = P + j\,Q \qquad [\text{VA}]$$

where $\boldsymbol{U}_{\mathbf{RMS}}$ and $\boldsymbol{I}_{\mathbf{RMS}}$ are the phasors of effective values of the voltage $\boldsymbol{U}_{\text{S}}$ and current $\boldsymbol{I}$

$$\boldsymbol{U}_{\mathbf{RMS}} = U_{\text{RMS}}\angle\alpha_{\text{U}} = \frac{U_{\text{m}}}{\sqrt{2}}\angle\alpha_{\text{U}}$$

$$\boldsymbol{I}_{\mathbf{RMS}} = I_{\text{RMS}}\angle\alpha_{\text{I}} = \frac{I_{\text{m}}}{\sqrt{2}}\angle\alpha_{\text{I}}$$

and $\boldsymbol{I}_{\mathbf{RMS}}^{*}$ is the conjugate of $\boldsymbol{I}_{\mathbf{RMS}}$

$$\boldsymbol{I}_{\mathbf{RMS}}^{*} = \frac{I_{\text{m}}}{\sqrt{2}}\angle -\alpha_{\text{I}}$$

Thus

$$\boldsymbol{S} = \frac{U_{\text{m}}}{\sqrt{2}}\angle\alpha_{\text{U}} \cdot \frac{I_{\text{m}}}{\sqrt{2}}\angle -\alpha_{\text{I}} = 947.82\angle -18.59° = 898.36 - j302.17\ \text{VA}$$

The average power is the real part of $\boldsymbol{S}$ => $P = 898.36$ W

The reactive power is the imaginary part of $\boldsymbol{S}$ => $Q = -302.17$ VAr

Another way to get these powers is

$$P = U_{\text{RMS}}\, I_{\text{RMS}} \cos\,(\alpha_{\text{U}} - \alpha_{\text{I}}) = U_{\text{RMS}}\, I_{\text{RMS}} \cos\,\varphi$$

$$Q = U_{\text{RMS}}\, I_{\text{RMS}} \sin\,(\alpha_{\text{U}} - \alpha_{\text{I}})$$

where $U_{\text{RMS}} = \dfrac{U_{\text{m}}}{\sqrt{2}}$ and $I_{\text{RMS}} = \dfrac{I_{\text{m}}}{\sqrt{2}}$

The MATLAB program for solving this task is

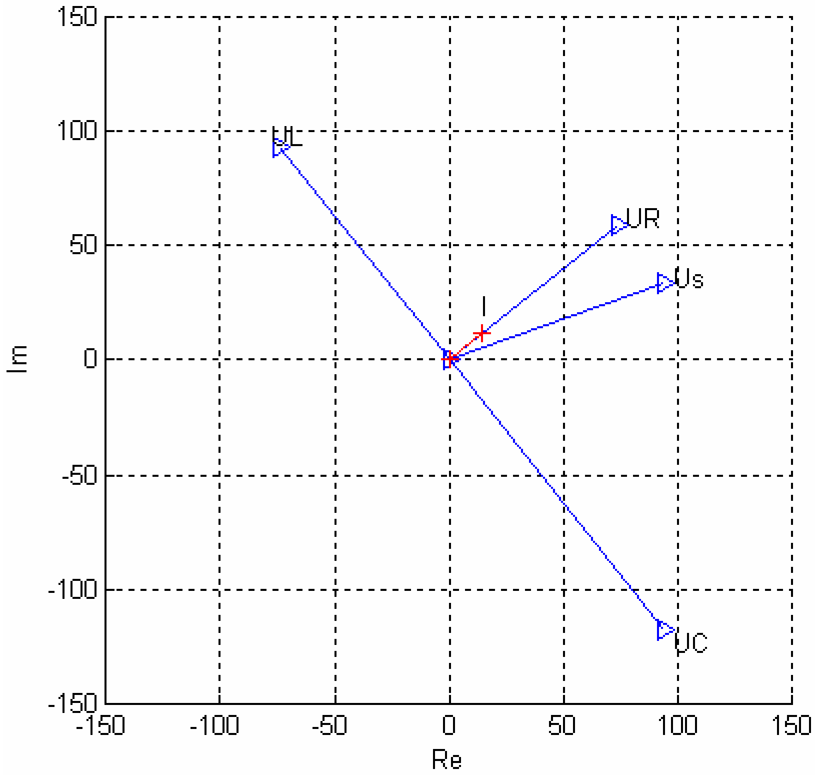
MATLAB Script

```
clear; clc;
% this program computes variables of RLC circuit
% and plots the phasor diagram
R = 5;        % ohms
L = 20e-3;   % H
C = 400e-6; % F
us_max = 100;   % V
us_ang = 20;    % angle in degrees
w = 314;
% complex representation of the source voltage
Us=us_max*exp(j*us_ang*pi/180);
% complex impedances
ZR=R; ZL=j*w*L; ZC=1/(j*w*C); Z=ZR+ZL+ZC;
% current and voltages
I=Us/Z; UR=ZR*I; UL=ZL*I; UC=ZC*I;
% RMS phasors
U_RMS=Us/sqrt(2); I_RMS=I/sqrt(2);
disp('The complex power is'); S=U_RMS*conj(I_RMS)
disp('The average power is'); P=real(S)
disp('The reactive power is'); Q=imag(S)
disp('The power factor is'); pf=cos(angle(S))
% phasor diagram
line([0 real(UR)],[0 imag(UR)],'marker','>');
text(1.05*real(UR),1.05*imag(UR),'UR');
line([0 real(UL)],[0 imag(UL)],'marker','>');
text(1.05*real(UL),1.05*imag(UL),'UL');
line([0 real(UC)],[0 imag(UC)],'marker','>');
text(1.05*real(UC),1.05*imag(UC),'UC');
line([0 real(Us)],[0 imag(Us)],'marker','>');
text(1.05*real(Us),1.05*imag(Us),'Us');
line([0 real(I)],[0 imag(I)],'marker','+','color','red');
text(real(I),2*imag(I),'I');
axis(1.5*us_max*[-1 1 -1 1]);
xlabel('Re'); ylabel('Im');
grid on; axis square;
```

The results obtained from MATLAB are

```
The complex power is

S =

  8.9836e+002 -3.0217e+002i

The average power is

P =

  898.3630

The reactive power is

Q =

 -302.1704

The power factor is

pf =

    0.9478
```

The phasor diagram obtained from MATLAB is



RH 2007