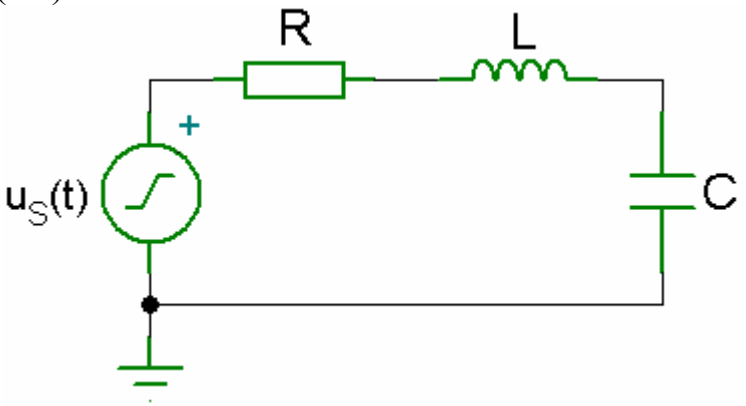
Resonance

Example

For the circuit shown below with using MATLAB plot the frequency dependency of the current magnitude and find the resonance frequency ω_r and the quality factor Q_r when R = 5 Ω, L = 20 mH, C = 400 μF and $u_S(t) = 100 \sin(\omega t)$ V.



RLC Circuit

Solution

Using complex frequency representation, for the voltage source $u_S(t)$ the phasor is $\boldsymbol{U_S}$ = 100 e $^{j\ 0°}$ = $100\angle 0°$. For the components R, L, C the complex impedances are

$$\boldsymbol{Z_R} = R$$

$$\boldsymbol{Z_L} = j\,\omega L$$

$$\boldsymbol{Z_C} = 1 / (j\,\omega C)$$

The total complex impedance is

$$\boldsymbol{Z} = \boldsymbol{Z_R} + \boldsymbol{Z_L} + \boldsymbol{Z_C}$$

The current is

$$\boldsymbol{I} = \frac{\boldsymbol{U_S}}{\boldsymbol{Z}}$$

The minimum value of $\boldsymbol{Z}$ exists just on condition that the imaginary part of $\boldsymbol{Z}$ equals zero:

$$\omega L - \frac{1}{\omega C} = 0$$

From the equation mentioned above, we get the resonance frequency

$$\omega_r = \frac{1}{\sqrt{LC}} = 353.55 \text{ rad/s}$$

The quality factor for the given circuit is

$$Q_r = \frac{\omega_r L}{R} = 1.4142 \ \ [1]$$

The MATLAB program for solving this task is

MATLAB Script

```
clear; clc;
% this program computes resonance of RLC circuit
R = 5;       % Ohms
L = 20e-3;   % H
C = 400e-6; % F
Us = 100;    % V
% vector of frequencies logarithmically equally spaced
% 1000 points between decades 10^0 and 10^5
w=logspace(0,5,1000);
% complex impedances
ZR=R; ZL=j*w*L;
ZC=1./(j*w*C); Z=ZR+ZL+ZC;
% current and voltages
I=Us./Z; UR=ZR.*I; UL=ZL.*I; UC=ZC.*I;
disp('The resonance frequency is'); wr=1/sqrt(L*C)
disp('The quality factor is'); Qr=wr*L/R
% graph
semilogx(w,abs(I),'linewidth',2);
xlabel('\omega [rad/s]'); ylabel(current magnitude [A]');
grid on;
```

The results obtained from MATLAB are

```
The resonance frequency is

wr =
  353.5534

The quality factor is

Qr =
    1.4142
```

The frequency dependency of the current magnitude obtained from MATLAB is